FEDERAL UNIVERSITY OF ITAJUBÁ

MECHANICAL ENGINEERING INSTITUTE

Optimization Methods

5th Exercise List

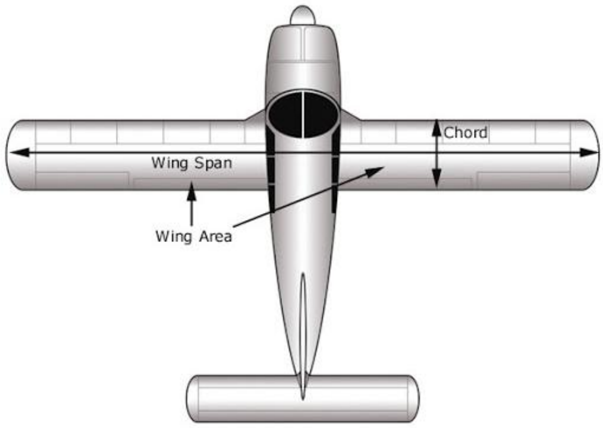
Chapter 5 – Nonlinear Programming II

Question 1. Find the minimum of the Rosenbrock function $f(x_1, x_2) = 100\left(x_2 - x_1^2\right)^2 + \left(1 - x_1\right)^2$, starting from the initial point $\mathbf{X}_1 = \{2.0\ 2.0\}^T$ using the following methods (develop your code in Scilab®, Python, or Matlab®):

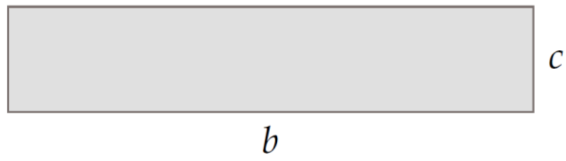
a) Steepest descent
b) Conjugate gradient (Fletcher-Reeves)
c) Newton

Plot the function, determine the graphical solution, compare and discuss the result of all methods using a table, considering a maximum number of iterations $N_{max} = 10$ for all methods. Calculate the error of each method in relation to the global optimum.

Question 2. The goal is to optimize the wing of an aircraft by altering its span and chord. Generally, we would add many more design variables to a problem like this, but we are intentionally limiting it to a simple two-dimensional problem so that we can easily visualize the results. Instead of minimizing drag, we want to minimize the required power, taking into account drag as well as propulsion efficiency, which depends on speed. The shape of the wing can be parameterized by the span (b) and chord (c) as shown in the Figure, so that $\mathbf{X} = [b, c]^T$.



(a)



(b)

Figure – Modeling of the rectangular wing according to two decision variables.

The following paragraphs describe a basic performance estimate and methodology for a low-speed aircraft. The implementation may not seem very important for optimization, however, the physics is important for our purposes, and the practice of translating equations and concepts into code is an important element in the formulation of optimization problems in general.

In level flight, the aircraft must generate enough lift to match the required weight:

$$L = W \tag{1}$$

And here we will assume that the total weight consists of a fixed aircraft and payload weight W_0, and a component of the weight that depends on the wing area S:

$$W = W_0 + W_S S \tag{2}$$

The wing can produce a certain lift coefficient C_L, so we must make the wing area S large enough to produce sufficient lift. Using the definition of the lift coefficient, the total lift can be calculated as:

$$L = qC_L S \tag{3}$$

where q is the dynamic pressure:

$$q = \frac{1}{2}\rho v^2 \tag{4}$$

Considering a rectangular wing, the wing area can be calculated from the span *b* and chord *c* as:

$$S = bc \qquad (4)$$

The drag of our aircraft consists of two components: viscous drag and induced drag. The viscous drag can be approximated as:

$$D_f = kC_f qS_{wet} \tag{5}$$

For a fully turbulent boundary layer, the friction coefficient C_f can be approximated as:

$$C_f = \frac{0.074}{\mathrm{Re}^{0.2}} \tag{6}$$

In this equation, the Reynolds number is based on the wing chord, $Re = \rho vc/\mu$. he form factor k accounts for pressure drag effects. The wetted area, Swet, is the area over which friction drag acts, which is a bit more than twice the planform area.

$$S_{wet} = 2.05S \tag{7}$$

Induced drag is defined as:

$$D_i = \frac{L^2}{q\pi b^2 e} \tag{8}$$

where e is the Oswald efficiency factor. The total drag is the sum of induced and viscous drag, $D = D_i + D_f$.

Our objective function, the power required by the engine for level flight, is:

$$f(b,c) = \frac{Dv}{\eta} \tag{9}$$

where η is the propulsive efficiency. We assume our electric propellers have a Gaussian efficiency curve (real efficiency curves are not very Gaussian, but this is simple and will suffice for our purposes):

$$\eta = \eta_{\max} \exp\left[\frac{-(v-\bar{v})^2}{2\sigma^2}\right] \tag{10}$$

In this problem, the lift coefficient is provided. Therefore, to satisfy the lift requirement (Eq. 1), we need to calculate the speed using Eq. 4 and Eq. 3 as:

$$v = \sqrt{\frac{2L}{\rho C_L S}} \tag{11}$$

The parameters for this problem are provided as:

Parameter	Value	Unit	Description
ρ	1.2	kg/m³	Air density
μ	$1.8\times(10^{-5})$	kg/(m.s)	Air viscosity
k	1.2	-	Form factor
C_L	0.4	-	Lift coefficient
e	0.80	-	Oswald efficiency factor
W_0	1000	N	Fixed aircraft weight
W_S	8.0	N/m²	Weight per area
η_{max}	0.80	-	Peak propulsive efficiency
$\overline{v}$	20.0	m/s	Flight speed at peak propulsive efficiency
σ	5.0	m/s	Standard deviation of the efficiency function

Formulate the optimization problem (statement) and choose one of the methods discussed in the Chapter to solve this problem. Justify the choice of the method and discuss the result obtained.

Note that there are no structural considerations in this problem, so the resulting aircraft wing profile may have a not very realistic ratio. This emphasizes the importance of carefully selecting the objective and including all relevant constraints.